Jaccard’s Heel: Radex Models of Criminal Behaviour are Rarely Falsifiable When Derived Using Jaccard Coefficient

Abstract

Purpose. This article considers whether the modular facet of popular 'radex' models of offender behaviour is falsifiable or a statistical inevitability when using Jaccard coefficient, as evidence from other domains suggests.

Method. Data equivalent to that examined in previous papers, and artificial data varying on 4 parameters, were examined using the conventional procedure of deriving Jaccard coefficients and submitting these to a Smallest Space Analyses (SSA-I). The parameters were number of variables, number of cases, highest frequency of variable occurrence, and distribution of occurrences. Evidence of a modular pattern in each SSA-I solution was assessed using one qualitative and two quantitative measures.

Results. When variables were free to occur in more than 50% of cases, none of the Jaccard-based SSA-I solutions supported the null hypothesis of no modular facet. This contrasts equivalent analyses using Yules Q, where 95.7% of the solutions supported the null hypothesis. When variables were restricted to occur in less than 50% of cases, the number of solutions supporting the null hypothesis changes to .003% and 78%, respectively. Analyses of the artificial data found that reducing the number of variables in a Jaccard-based solution increased the likelihood of supporting the null hypothesis, which suggests that these solutions are structured by variable occurrence (i.e., frequency) rather than variable co-occurrence.

Implications. Research using Jaccard coefficient to measure co-occurrences among behaviours should not claim that the modular facet of their radex model is an empirical finding. Unfortunately, this is many of the existing publications.

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Over the last decade, articles in *Legal and Criminological Psychology* and related periodicals have proposed models that differentiate the ways in which offenders commit their crimes (e.g., Canter, 2000; Canter & Fritzon, 1998; Häkkänen, Puolakka, & Santilla, 2004; Mokros & Alison, 2002; Santtila, Häkkänen, Alison, & Whyte, 2003). When presented graphically, these models are characterised by a circular ordering of wedge-like regions that extend outward from a common origin. Each of these regions denotes one of the interpersonal styles that offenders are predicted to bring to their crimes. The circular structure containing the regions is known as a radex.

One aspect of these models that has recently attracted attention is the conceptual change associated with movement from the common centre of the regions to their periphery (i.e., from the middle to the outside edge of the radex model). This distinction is known as the 'modular facet' (Guttman, 1954) and it has long been regarded as having empirical and theoretical importance in other areas of social science (Donald, 1985; Maslovaty, Marshall, & Alkin, 2001; Schlesinger & Guttman, 1969; Taylor & Donald, 2007). More recently, it has been presented as a valuable empirical finding in models of criminal behaviour (Almond, Duggan, Shine, & Canter, 2005; Canter, Giles, & Nicol, 2004; Canter & Fritzon, 1998; Santtila et al., 2003; Youngs, Canter, & Cooper, 2004). However, as addressed in this article, there is reason to believe that evidence of the modular facet in models of criminal behaviour is inevitable and not of empirical value. This is because of the way in which these models conceptualise the modular facet, and because of the unique methodology that is used to test these models.

This paper begins by examining the theoretical nature of the modular facet and the conditions necessary for it to be falsifiable in studies of criminal behaviour. Using the methodology typical of research in the area, the paper then examines the likelihood of observing evidence that falsifies the hypothesis of a modular facet under different conditions. This analysis considers both data analogous to that examined in previous research (to evaluate the value of previous evidence) and data that varies on a range of characteristics (e.g., the shape of frequency distribution). The results have implications for existing and future investigations of radex models of criminal behaviour.

Multivariate Models of Offender Behaviour

The empirical basis of most radex models of criminal behaviour (and the ones considered here) is an analysis of dichotomous data that records the occurrence (1) or non-occurrence (0) of a set of offender behaviours (e.g., blindfold victim, punch the victim) across different crime scenes. These data provide a way of examining the kinds of behaviours that regularly occur together in crimes. A radex model gives a conceptual interpretation of these data by identifying groups or 'regions' of highly co-occurring behaviours that instantiate a single explanation for offending. For example, a number of authors predict that an offender's desire for social contact and intimacy underpins the offence of rape (e.g., Marshall, 1989). If true of some offenders, one would expect to find a discrete region of co-occurring behaviours in the model that reflect this approach (e.g., revealing personal information, attempts to converse with the victim). Typically, a radex model contains several such regions of behaviours, which reflects the fact that different offenders bring different interpersonal styles to their crimes.

In a typical study of criminal behaviour, the extent to which behaviours co-occur is measured using Jaccard association index; a coefficient calculated as the number of occasions two behaviours co-occur over the cases as a proportion of the occasions when at least one of the behaviours occurs. These co-occurrences are then examined using a multidimensional scaling method (e.g., Smallest Space Analysis, SSA-I, Borg & Shye, 1995) that depicts behaviours as points in a notional 'space,' where the points are arranged in such a way that they appear closer together the more often two behaviours co-occur across the crimes. This graphical presentation may be examined for evidence of regions of co-occurring behaviours, where these regions are underpinned by a theoretical explanation for why the behaviours occur together.

Figure 1 presents a schematic example of the type of model that results from this form of analysis (for the original see Taylor, Bennell, & Snook, 2002). The model in Figure 1, like all radex models, is structured by two theoretical distinctions known as facets. The first facet divides the space into wedge-like regions that reflect the different qualities or styles of offender behaviour (in this case, criminal controlling, pseudo-intimate, and violence). The second facet relates to movement from behaviours that are shared by all offence styles, and therefore found in the middle of the model, to behaviours specific to one style, which are found at the periphery of the model. The model adopts the geometric shape described as a radex because of the intersection of these two facets. Both must exist for a radex structure to emerge. Equally, it must be possible to derive data that do not support each of the facets, if the radex model is to be described as falsifiable.

Insert Figure 1 about here

The Modular Facet in Radex Models of Criminal Behaviour

The modular facet has received significant attention in studies of criminal behaviour in recent years (e.g., Almond, Duggan, Shine, & Canter, 2005; Canter, Giles, & Nicol, 2004; Santtila et al., 2003). This attention is geared towards providing a theoretical account of the difference between behaviours found at the centre of the model and behaviours found at the periphery of the model. As Canter and Wentink (2004) explain, "The contours of frequency have also been found in a number of other studies of criminal actions, indicating a lawful consistency to this empirical finding" (p. 511). In the majority of studies, this empirical finding is posited as relating to the salience of criminal behaviour, with movement from the centre to the periphery of the model associated with change from general to unique behaviours. For example, Canter (2000) defines movement from the centre to periphery as change through "Typicality", "Behavioural pattern", "Modus operandi", and "Signature". In other papers, such as Canter, Bennell, Alison, and Reddy (2003), the modular facet is proposed to reflect increasing levels of violation, from sexual to physical to personal levels of violation. This second example is distinct from the first because of its effort to articulate an order to the kinds of violation observed, though it arguably does so without providing an account for why 'personal' is the highest level of violation (a point addressed below).

As well as attempting to map conceptual meaning onto the modular facet, researchers are using its occurrence or non-occurrence in data to test other theories of criminal behaviour. For example, Canter, Alison, Alison, and Wentink (2004) refute the FBI's organised/disorganised typology of serial murder on the basis that many of the behaviours associated with the 'organised type' occur in the centre of their SSA-I solution. This, they argue, indicates that organised behaviours are central to all offences and so not unique to a subgroup of offenders as the FBI typology suggests. In a similar way, Salfati and Taylor (2006) use evidence of the modular facet to challenge the value of treating sexual homicide and rape as distinct crimes. Their analysis of sexual homicide and rape cases in a single model continued to show a distinct circular ordering of behaviours, shaped by a modular facet of frequency. They argue that this suggests that sexual homicide and rape are better understood as different emphases of a single type of criminal activity, which they call sexual assault. In both cases, the conclusions drawn from the findings represent appropriate interpretations of the interrelationships among behaviours. However, they each assume that the patterning from the centre to periphery of the model results from an empirical, testable pattern in the data. For example, in the Canter et al. (2004) study, the assumption is made that 'organised' behaviours appear towards the middle of the plot for conceptual reasons, and not simply because there are more 'organised' offenders in the data. As Canter (2000) argues, "The model of behavioural salience is a refutable hypothesis because it is possible that distinct subgroups of actions could occur in any class of crime which, whilst frequent, were typically associated with distinct sets of rarer actions. In such a case the concentric circles that make up the radex would not be found" (pp. 34-35).

Confusing Conceptual Value with Statistical Artefact

What if these findings were not the result of a unique pattern of co-occurrences among behaviours but the product of some other aspect of the data or analysis? While it is common to view the modular facet as a refutable hypothesis, there is reason to question whether or not this view is valid given the unique way in which existing studies analyse behaviour. This question is prompted by indirect evidence showing that Jaccard coefficient is inflated by the frequency of occurrence of a variable (e.g., a behaviour; Chung & Lee, 2001; Jackson, Somers, & Harvey, 1989; see the next section for a discussion of why this may occur). The implication of this evidence is that frequently occurring behaviours will end up correlating highly with other behaviours in the analysis, which will result in a tendency for these behaviours to fall toward the middle of the multidimensional scaling solution (where their average proximity to other behaviours is minimal). If this tendency impacts significantly on the overall placement of behaviours in a solution, then the modular facet becomes something determined by behaviour frequency rather than behaviour co-occurrence. Thus, what may appear as a substantive finding about offenders' interpersonal tendencies, such as the absence of regions reflecting organised and disorganised styles (Canter et al., 2004), may in fact be confounded by differences in the frequency of occurrence of the variables (i.e., there were simply more 'organised' offenders in the data).

Any link between variable frequency and the modular facet is critical to research on criminal behaviour because studies in this area often relate the modular facet to changes in frequency. This approach to theorising about the modular facet is unique to studies of criminal behaviour. In other domains, modular facets typically capture changes in the conceptual meaning of variables rather than changes in their statistical value. For example, in their analysis of intelligence tests, Guttman and his colleagues (Guttman, 1954; Guttman & Levy, 1980; Schlesinger & Guttman, 1969) found support for a modular facet that reflected differences in the type of skill required to answer a test item correctly. Their facet differentiated rule-inferring tasks, which occurred at the centre of their model, rule-application tasks, which occurred in the middle ring of the model, and rule-learning tasks, which occurred on the peripheral ring of their model. Thus, their modular facet was defined in terms of qualitative difference and not in terms of a statistical difference (e.g., the number of times the item was answered correctly). It was clearly tied into what Guttman describes as a “common range”, namely, a single, theoretical construct to which all of the variables can be seen to relate (i.e., intelligence)(Levy, 2005). Of course, in the absence of a strong theoretical account it is reasonable for an exploratory study to heuristically examine the relationships among offender behaviours in order to develop a basis for subsequent research. But such exploration is a first iteration and its findings do not support claims of “lawful consistency.” Indeed, Guttman proposed a systematic way of stating radex models—known as the mapping sentence—with the explicit purpose of ensuring that research progressed cumulatively beyond initial exploration (for more details, see Borg & Shye, 1995).

The traditional conceptualisation of the modular facet has at least two implications for radex models of criminal behaviour. The first is that any hypothesised modular facet should be linked to a theory of criminal behaviour and not simply to a description of the pattern of frequencies. Unfortunately, this underlying common construct is not always made clear in published models. The second is that it must be possible to derive solutions that do not support the hypothesised concentric circles of frequencies. This implication precedes the first because the model must be falsifiable if it is to have any scientific value as a theoretical explanation. Thus, while both concerns deserve attention, it is the second methodological implication that is the focus of this paper.

Why is Jaccard a Special Case?

At the root of our concerns about the inevitability of current models is the Jaccard coefficient and its unique way of measuring behaviour co-occurrences. Traditional studies of radex models measure the associations among behavioural variables using coefficients such as Pearson's correlation coefficient (Schlesinger & Guttman, 1969) or Guttman's Mu coefficient (Ben-Shalom & Horenczyk, 2003). These coefficients may be considered symmetrical measures in the sense that they treat the joint non-occurrence (or joint low frequency) and joint occurrence (or joint high frequency) of two variables as equivalent instances of association. Two behavioural variables occurring infrequently within the crimes may be measured as having the same degree of association as two behaviours occurring frequently within the crimes. The value of the coefficient is therefore independent of the two behaviours' frequency of occurrence.

In contrast, Jaccard coefficient is an asymmetrical measure in the sense that it does not value joint non-occurrences in the same way as joint occurrences. Specifically, a Jaccard coefficient is computed as the number of occasions when both behaviours occur (often denoted as cell A), divided by the number of occasion when at least one, or both, behaviours occur (often denoted as cells A + B + C).[1] Thus, the number of occasions when neither of the behaviours occurs is not considered when computing the coefficient. This approach minimises a researcher's reliance on non-occurrences, which is useful in studies of criminal behaviour because it is difficult to know whether the behaviour did not occur, or whether the behaviour was simply not recorded as occurring. However, the consequence of taking this approach is that the value of the coefficient is no longer independent of variable frequency. As the frequency of a behaviour's occurrence increases, so too does its chance of co-occurring with another behaviour. Since these co-occurrences determine the maximum value of the Jaccard coefficient (i.e., their sum is the numerator), the higher the frequency of occurrence of a behaviour, the higher on average the behaviour's association with other behaviours.[2]

The higher correlations that result from behaviours occurring frequently may well influence where these behaviours appears in the multidimensional scaling analysis. It is reasonable to predict that they will fall towards the centre of a multidimensional scaling solution, since this is the point at which they are nearest to (and therefore represented as most associated with) the most behaviours, with other less frequent behaviours positioned outside of the high frequency "core." In situations where this is true, the radex model becomes less of an empirical phenomenon and more of a methodological artefact, since one of its two underlying facets becomes un-testable. Specifically, under these conditions, it will be impossible to derive support for modular facets that are independent of the frequency of occurrence of behaviours, and impossible to falsify those that do correspond with the frequency of occurrence of behaviours.

This paper examines the extent to which modular facets are falsifiable when using Jaccard coefficient.[3] The data examined is designed to vary on four parameters that may impact on the likelihood of observing non-frequency related modular facets. For example, the highest frequency of occurrence of behaviour is varied to determine whether capping this frequency may make empirical testing possible. Similarly, the number of behaviours in the model is varied to determine whether or not having more variables reduces the impact of frequency on the model structure. The results will clarify the conditions under which it is possible to falsify the modular facet of a radex model of criminal behaviour.

Method

Data

To determine the likelihood of observing a modular facet when using Jaccard coefficient, matrices of dichotomous data were produced and analysed. These data were generated using the random number function of the Perl programming language. The program generated random dichotomous data in which a 1 represented the occurrence of a variable and a 0 represented the non-occurrence of a variable. The data produced varied on four features: i) number of variables; ii) number of cases; iii) highest frequency of occurrence of a variable; and iv) distribution of the frequencies of occurrence across the variables. A manipulation of these four features made it possible to produce a diverse range of data, including data whose form (i.e., number of variables, cases, and distribution of frequencies) was equivalent to that analysed in previous research. For each form of data, we produced multiple data matrices (100 matrices for each "evaluation" data and 50 matrices for each "exploratory" data; see below). If the modular facet is an alterative hypothesis whose (non-)occurrence is independent from behaviour frequency, then absence of a modular facet (i.e., the null hypothesis) should be observed in the majority of these data sets (e.g., in approximately 45 of 50 data sets when adopting a liberal $\alpha = .10$).

The data generated using the Perl program comprised of two sets. The first "evaluation" set consisted of data that replicated the form of the data analysed in three previous studies. Data with 27 variables and 112 cases, with a maximum frequency of occurrence of 92 (82%) and minimum frequency of occurrence of 4 (4%), was generated to examine the likelihood of finding a modular facet in data equivalent to that examined in Canter et al. (2003). Data with 42 variables and 175 cases, with a maximum frequency of occurrence of 147 (84%) and minimum frequency of occurrence of 4 (2%), was generated to examine the likelihood of finding a modular facet in data equivalent to that examined in Canter and Fritzon (1998). Data with 33 variables and 66 cases, with a maximum frequency of occurrence of 55 (83%) and minimum frequency of occurrence of 5 (8%), was generated to examine the likelihood of finding a modular facet in data equivalent to that examined in Canter and Heritage (1990). These examples were chosen for comparison because they consider different types of crime, vary in the number of cases they analyse, and represent studies distributed across 14 years of research in the field. They also provide the information needed to produce equivalent data, they present evidence of a modular facet, and they discuss its implications.

The second "exploratory" set covered a wide range of data forms in order to determine the conditions under which a modular facet may or may not be falsified. In this group, data were manipulated to: i) include 20, 30, 40, or 50 variables; ii) to be comprised of 50, 100, 150, or 200 cases; iii) to have variables whose highest frequency of occurrence was 100%, 80%, 60%, or 40% of the number of cases; and, iv) to have a skew in the distribution of frequencies of -1.0, -0.5, 0.0, +0.5, and +1.0, as measured by the skewness statistic (Howell, 1997, p. 27; Pearson, 1895). The manipulation of each data feature was independent of the manipulation of the other features, such that, in a similar way to a factorial design, all possible combination of data features was examined. In all cases, the lowest frequency of occurrence for a variable was 5% of the number of cases, since this cut-off is often suggested as the minimum frequency necessary for a variable to be useful at differentiating among offenders (Canter & Heritage, 1990).[4]

Procedure

Consistent with previous research, data were analysed using Jaccard coefficient and the non-metric multidimensional scaling procedure known as Smallest Space Analysis (SSA-I; Lingoes, 1973). Specifically, for each data set, the co-occurrences among variables (i.e., the simulated behaviours) were measured using Jaccard coefficient, and a matrix of these coefficients derived as a measure of the co-occurrence of each variable with every other variable. This matrix was then submitted to a SSA-I in three dimensions. SSA-I presents the rank order of the coefficients visually as distances among points that represent the variables. The higher the association between two variables (i.e., the higher the coefficient), the closer the points representing them will appear on the SSA-I plot. The result of this presentation is a constellation of points that allows an assessment of the relative co-occurrences among variables. This SSA-I solution may then be examined for predicted groups or trends in the behaviours, including evidence of a modular facet (for more details see Donald, 1985; Donald & Cooper, 2001; Taylor, 2002a; Taylor & Donald, 2004).

A general indication of how well the SSA-I configuration represents the co-occurrences of behaviours is provided by the coefficient of alienation (Borg & Shye, 1995). The smaller the coefficient of alienation, the better the correspondence between the rank order of Jaccard coefficients in the matrix and the rank order of distances among the variable points in the SSA-I configuration. Existing studies often consider a coefficient of alienation below .2 as an indicator of a useful solution (Salfati & Taylor, 2006). In the analysis that follows, the SSA-I solutions for the evaluation data all had a coefficient of alienation lower than .23 (*M* = .17, *SD* = .03). The SSA-I solutions for the exploratory data all had a coefficient of alienation lower than .27 (*M* = .16, *SD* = .04).

Finally, to determine whether the observations were particular to the use of Jaccard coefficient, each of the three evaluation data sets were subjected to an equivalent analysis in which the associations were measured using Yule's Q coefficient. Unlike Jaccard coefficient, Yule's Q considers joint non-occurrence as an indication of an association, such that two infrequent variables may be associated with a high coefficient value.

Assessing the Structure

There are a number of different ways to assess an SSA-I solution for evidence of a modular facet. Three complementary measures were used in this research. In each case, the measure was used to assess the extent to which there was a modular pattern to the frequency of occurrence of the variables presented in the first and second dimensions of the SSA-I solution.

Visual interpretation. One widely used approach to examining an SSA-I output is visual interpretation of the configuration. This approach involves identifying a series of concentric rings from the middle to the periphery of the SSA-I solution, where the frequency of occurrence of the variables located within a ring fall in a range that is less than the frequency range of inner rings (i.e., those located closer to the middle of the solution) and greater than the frequency range of outer rings (i.e., those located further toward the periphery of the solution). This results in a series of rings reflecting decreasing variable frequency with movement from the centre of the SSA-I configuration. The dotted rings in Figure 1 show this form of assessment schematically.

This approach was replicated here by overlaying the frequency of occurrence of the variables on their representative SSA-I points on the configuration. The first author then attempted to draw four concentric rings on the configuration such that each ring contained only variables whose frequency of occurrence was equal to or less than the ring immediately inside it. This approach therefore aimed to create five concentric regions. In line with previous research, a configuration on which the four concentric rings were drawn with minimal error was considered support for the modular facet.

Regionalisation. The ability to fit concentric regions to the frequency groupings was also assessed using an existing measure of regionality known as *v* (Amar, 2001). The *v* measure indicates the extent to which it is possible to draw regions that divide the variables into their predicted levels. It varies between .00 and 1.00, where .00 indicates that there is no evidence of the predicted facet in the configuration and 1.00 indicates that all variables may be regionalised as predicted. Since some variables will fall into the correct regions by chance, the true minimum of *v* is above .00. The actual baseline value of *v* (i.e., chance *v*) is best derived through a permutation test (Amar & Cohen, 2001). This involves repeatedly calculating *v* for the configuration under circumstances where the variables are randomly assigned to the levels of the modular facet. This permutation of the levels derives the range of *v* that could be observed within the configuration. The average of this range is the *v* that would be expected by chance. Thus, the original *v* minus the chance *v* provides a comparable measure of how much a modular facet is found within a solution relative to a standard 0.0 baseline. This measure is referred to below as corrected-*v*.

The corrected-*v* solutions were examined in the same way as our visual interpretations. Specifically, the variables were grouped into five frequency bands and *v* used to assess the extent to which these bands formed a modular facet within the configuration. For each data set, the variables were ranked by descending frequency of occurrence and then moved down the ordered list assigning an equal number of variables into one of five discrete levels. These levels were then adjusted so that two or more variables with the same frequency of occurrence were assigned to the same level. This was achieved while maintaining, as far as it was possible, the same number of variables in each of the five levels. The extent to which the position of variables within the solution corresponded to these grouping was then measured using corrected-*v*.

Montonicity. While modular facets are often demonstrated using concentric regions, the essence of the modular hypothesis is a conceptual change from the centre to the periphery of the configuration. Consequently, a more general way of testing the inevitability of a frequency-based modular facet is to measure the relationship between the frequency of occurrence of the variables and their distance from the configuration centre. This relationship was measured by correlating behaviour frequency with a measure of the behaviours distance from the geometric centre of the multidimensional scaling configuration (a measure given by the multidimensional scaling analysis software). The non-parametric Gutttman's Mu coefficient was used to measure this relationship (Guttman, 1986). This was to ensure that the relationships were assessed according to the relative distances away from the centre of the configuration (i.e., lower frequencies should be associated with larger distances), and not on the basis of absolute distance, which may vary as a function of extrinsic differences such as the number of variables being examined (cf. Taylor, 2002b). Because, like Amar's *v*, Guttman Mu may also not have a true minimum of 0.0 when used in this way, permutations equivalent to that described above were run to ascertain a 'true' baseline. This measure is referred to below as corrected-Mu. If the placement of variables on the SSA-I configuration is not determined by variable frequency, then the correlation between frequency and distance from origin, as measured by corrected-Mu, will not differ significantly from chance.

The Guttman Mu measure has at least two advantages over the other methods for this study. First, the calculation of the correlation is easy to automate, thereby allowing rapid testing of data that varies on a range of parameters while avoiding the potential bias of visual interpretations. Second, by considering only distance from the origin of the solution, the approach makes it unnecessary to make a choice about the number of concentric rings (contours) that are superimposed to test the modular facet. This is important because the number of contours should be determined by theoretical distinctions. The number chosen within the analysis may have a significant impact on the value of *v* and subsequent conclusions about the existence of otherwise of the modular facet. Thus, while Amar's *v* is the more traditional quantitative measure of assessment, Guttman's Mu guards against this possible bias.

Results

Analysis of Evaluation Data

Visual interpretation. Six forms of data, each represented by 100 data sets, were examined using visual interpretation (i.e., the overlaying of four concentric rings where each ring contained behaviours whose frequency of occurrence was equal or less to the frequency of occurrence of behaviours in the more central regions). Three of these data forms matched the original publications of Canter and Heritage (1990), Canter and Fritzon (1998) and Canter et al. (2003). The three remaining data forms were equivalent to the original but for an adjusted highest frequency of occurrence of the variables. The reliability of categorisation was assessed by having the third author independently interpret 300 (50%) of the solutions. Agreement, measured using Pearson's interclass correlation coefficient averaged using Fisher's *r*-to-*z* and *z*-to-*r* transformations, was .72.

Insert Table 1 about here

Table 1 presents the mean number of errors encountered when four concentric rings were imposed on the solutions for each of the data sets. Because the mean number of variables not fitting the modular facet is likely a function of the number of variables being analysed, Table 1 also presents the average proportion of variables that were not consistent with the imposed regions. As can be seen from Table 1, the number of variables not fitting the imposed regions is approximately 2 when examining data equivalent to that published. This equates to approximately 5% of the variables on any one solution, which compares to the 80% (i.e., one in every five) that would be expected on a random solution. This proportion, and hence the degree of support for the null hypothesis, rises when the highest frequency of occurrence is constricted to lower frequencies. When the occurrence of a variable is restricted to 25% of the number of cases, the number of misplaced variables rises to 11%. This increase is consistent with the proposal that the asymmetric nature of Jaccard coefficient reduces the likelihood of finding support for the null hypothesis.

Regionalisation. Figure 2 shows the mean corrected-*v* as a function of data set and the maximum frequency with which variables could appear in the solution. The Figure uses solid markers to denote solutions derived using Jaccard coefficient and hollow markers for solutions derived using Yule's Q. The data points at approximately 83% on the x-axis (i.e., Highest Frequency of Occurrence) are mean corrected-*v* for data whose form is identical to that of the three published studies. These data therefore allow an assessment of the extent to which it would have been possible to falsify the modular facet supported in these studies. Because Figure 2 shows corrected-*v*, the x-axis (i.e., $v = 0.0$) is the average score expected if all of the data support the null hypothesis.

Insert Figure 2 about here

As can be seen from Figure 2, solutions derived using Jaccard coefficient do not fall close to the 0.0 baseline, as would occur when the majority of solutions support the null hypothesis. Of the 1,500 Jaccard solutions derived for Figure 2, 5 fell below 0.0 and 46 (.03%) were found to be within two Standard Deviations of 0.0 (i.e., 96.9% reach the $p < .05$ significance level).[5] All of the five cases that crossed 0.0 were solutions in which the variables were constrained to occur in no more than 25% of the total cases. Indeed, as shown on Figure 2, for each of the data sets there is a decreasing trend in corrected-*v* as the highest frequency of variable occurrence is restricted.

The Yule's Q data show a different pattern. Under conditions when the occurrence of variables was not restricted to more than 50% of the cases, 93.6% of the corrected-*v* fell within two Standard Deviations of 0.0. This is in line with what is to be expected when $\alpha = .05$. However, with the inclusion of cases where the range was restricted, the proportion of corrected-*v* within two Standard Deviations fell to 85.9%. This apparent increase in average corrected-*v* as the frequency range is constricted contrasts the observed decrease for the Jaccard based solutions, and is discussed below.

Monotonicity. Figure 3 presents the mean corrected-Mu coefficients for the evaluation data using the same format as Figure 2. As can be seen from Figure 3, when the potential confound of regioning is removed from the analysis, the difference in findings between Jaccard and Yule's Q based analyses becomes even more apparent. Specifically, for the Jaccard based solutions, corrected-Mu is almost at ceiling until the frequency of variable occurrence is restricted to less than 25% of the cases. Of the 1,500 Jaccard solutions derived for Figure 3, only one fell below 0.0 and five (0.03%) were found to be within two standard deviations of 0.0. Again, the Yule's Q data shows a different picture. Under conditions when the occurrence of variables was not restricted to more than 50% of the cases, 95.7% of the corrected-Mu fell within two Standard Deviations of 0.0. However, when cases where the range was restricted were include, the proportion of corrected-Mu within two Standard Deviations fell to 78%. As with the corrected-*v* analysis, there is a differentiation between decreasing corrected-Mu for Jaccard-based solutions and increasing corrected-Mu for Yule's Q solutions.

Insert Figure 3 about here

Analysis of Exploratory Data

To determine the viability of using corrected-Mu as the basis of assessing the exploratory data, the mean corrected-*v* were correlated with the corrected-Mu across all the evaluation data (N = 14). The resulting correlation, $r = .84$, suggests that corrected-Mu serves as an expedient method for assessing the extent of the relationship between frequency and modular facet, for the purposes of the exploratory data analysis.

Figure 4 shows the mean corrected-Mu coefficients for data varying on parameters of the number of variables, number of cases, highest frequency of variable occurrence, and skew of the distribution of occurrences. As can be seen from Figure 4, none of the data sets approach a corrected-Mu of 0.0, which would occur if the majority failed to find a modular facet (i.e., supported the null hypothesis). Indeed, of the 16,000 solutions that comprise the means shown on Figure 4, none were below 0.0 and only one was within two standard deviations of 0.0.

Insert Figure 4 about here

To determine what parameters have the greatest influence on the corrected-Mu, and therefore determine what aspects of the data most contribute to the inability to find support for the null hypothesis when using Jaccard, we regressed corrected-Mu on the four data parameters (i.e., number of variables, number of cases, highest frequency of occurrence, and distribution skew). A step-wise regression procedure was used in which a variable was included in the regression model when the change in $r > .10$, as is appropriate when N is large (Donald et al., 2005). The resulting model contained only one predictor, Number of Variables, which predicted about 75% of the variance found within corrected-Mu, $r = .86$, $F(1,15998) = 47{,}717.3$, $p < .01$ ($r = .89$ for all four predictors). The impact of the Number of Variables on corrected-Mu is easily observed in Figure 4, with step-like increments occurring in the set of average lines from the left-hand column (Number of Variables = 20) to the right-hand column (Number of Variables = 50).

Discussion

The radex model has been influential in the study of criminal behaviour, with publications spanning across two decades and a variety of crime types (Alison, Goodwill, Almond, van den Heuvel, & Winter, 2010). In recent years, studies of these models have begun to theorise about the conceptual distinction between behaviours at the centre and periphery of the model. This focus on the ‘modular facet’ is a healthy development for the field, not least because modular facets are significant components of theories of human behaviour in other domains (e.g., Donald, 1985). However, it also raises important questions about the conceptual and methodological conditions under which models of criminal behaviour may be tested. This paper examined the common method for deriving radex models of criminal behaviour, namely, an analysis of the co-occurrences among crime scene behaviours coded dichotomous and measured using Jaccard association coefficient. The findings overwhelmingly suggest that this approach does not provide a meaningful empirical test of the existence or otherwise of the modular facet. As a result, this approach does not provide a full test of radex models of criminal behaviour.

Two difficulties with the existing approach to examining offender behaviour were identified. The first relates to a concern about the conceptualisation of the modular facet, which in studies of criminal behaviour is often based on variable frequency rather than a conceptual distinction among the variables. Underlying this concern is the absence of a theoretical ‘common range’ that relates all of the behaviours to the domain being examined (Levy, 2005). The second relates to an almost complete failure to produce data that did not support a frequency-based modular facet. In tests of data produced to be equivalent to that published in three leading publications (Canter et al., 2003; Canter & Fritzon, 1998; Canter & Heritage, 1990), and in simulations that covered a wide range of possible data, only 52 instances in which the null hypothesis would have been supported were found. This frequency represents .002% of the tests run for this paper; a percentage that contrasts sharply with the 95% that would be expected to occur given a standard Type I error rate (α) of .05. In contrast, our analyses using Yule’s Q coefficient found only about 5% of the data sets containing modular facets correlated with the frequency of occurrence of the variables. This percentage is what would be expected under a $\alpha = .05$ error rate. Thus, unlike the Jaccard analyses, the modular facet in radex models derived from Yule’s Q coefficients does appear to be testable and not inevitably related to variable frequency of occurrence.

One interesting difference between the data produced from the Jaccard and Yule’s Q solutions was the change in support for the null hypothesis when the range of the frequency of occurrence of variables was restricted. In the Jaccard based solutions, a restriction in frequency range reduced the inevitability of a frequency-based modular facet. In the Yule's Q based solutions, this restriction led to an increase in the occurrence of frequency-based modular facets. Both of these findings are consistent with the explanation of the impact of variable frequency on asymmetric and symmetric measures of association. In the Jaccard case, reducing the variation in the frequency of occurrence of variables serves to reduce the extent to which variable frequency can influence the correlations among the variables. The result is SSA-I solutions that are less defined by variable frequency. In the Yule's Q case, reducing the variation in frequency of occurrence attenuates the coefficients.

A regression analysis confirmed the importance of frequency variation on the falsifiability of the radex. It revealed that the number of variables in the analysis, more than any other data variation, increased the average support observed for the modular facet across the evaluation data. A greater number of variables allows for more variation in the frequency of occurrence of variables, which in turn leads to frequency having a larger impact on variable co-occurrences. Thus, together with the other analyses, this finding suggests that it is almost impossible to empirically test a modular facet when using Jaccard coefficient, since variables will structure themselves around variable frequency of occurrence rather than variable co-occurrence. This statistical inevitability is clearly at odds the conceptual richness that is often attributed to the modular facet by the use of terms like "refutable" (Canter, 2000; p. 34) and "lawful consistency" (Canter & Wentink, 2004; p. 511).

So what are the implications of these finding for radex models of criminal behaviour? There is arguably a strict and relaxed answer to this question, depending on how one interprets the criteria for evidence of a radex model. A strict interpretation of the theoretical approach that underlies radex models suggests that evidence of both a polar facet (the groupings or “themes” of behaviour found in a radex model) *and* the modular facet must exist before a researcher can claim to have demonstrated a radex. This position recognises that radex models are structured by the intersection of the polar facet sectors and the modular facet’s dispersal of those sectors from the centre of the solution (Borg & Groenen, 2005). If this dispersal is due to a statistical inevitability, then it is impossible to determine whether or not the groupings themselves would emerge under conditions where the modulations were not inevitable. Thus, in this sense, the use of SSA-I with Jaccard coefficient cannot provide an empirical test of a radex model.

The difficulty with this position, aside from the non-trivial doubts it raises about previous results, is that it fails to acknowledge the regions of behaviour that emerge when using Jaccard. The emergence of these regions may, in part, be statistically independent of the modular facet. They may draw on a different set of structural relations among the behaviours that are not determined by frequency. Thus, a more progressive interpretation of this paper’s findings might be to accept that the modular facet is un-testable, but to view the variable inter-correlations as still providing insights into the co-occurring groups of behaviours that instantiate offender interpersonal style. With this relaxed position, it is possible to make a theoretical contribution while using Jaccard coefficient, so long as the testing of hypotheses is confined to tests regarding the co-occurrence of behavioural subgroups (i.e., the polar facet).

The validity of this less stringent position rests on the degree to which differences in the regioning of variables is independent from the modular pattern. The extent of this independence under Jaccard coefficient is unclear. It seems likely that fixing the modular structure of the SSA-I (the placement of behaviours from the centre to the periphery) will make the configuration sensitive to differences in behaviour co-occurrence. This is because fixing the approximate modular position (from centre to periphery) of a variable has the effect of transforming the configuration into a one-dimensional unfolding model that is joined at either end (i.e., a donut). In this situation, variables will reorder along the dimension according to their relative correlation, such that it only takes a small increment in the co-occurrence between two variables for those variables to be re-positioned next to one another on the polar dimension. In this scenario, it becomes difficult to know whether patters of co-occurrences around the configuration are substantively meaningful beyond small (non-significant) differences in the data. Overall, then, it appears prudent to avoid giving too much theoretical weight to radex models of criminal behaviour until ways of overcoming the Achilles' heel of Jaccard coefficient are developed.

Footnotes

[1] This calculation is often presented mathematically as $\frac{A}{A+B+C}$ where the letters refer to count data taken from a 2 x 2 representation of possible relationships:

		Variable Q	
		1	*0*
Variable P	*1*	A	B
	0	C	D

A = Variable P and Variable Q occur
B = Variable P occurs, Variable Q does not occur
C = Variable P does not occur, Variable Q occurs
D = Variable P and Variable Q do not occur

[2] It is also possible to show that the minimum value of a Jaccard coefficient for two variables differs as a proportion of each variable's frequency of occurrence. Specifically, the lower minimum increases as variable frequency increases beyond the point at which the sum of occurrence across the two variables is greater than the number of cases. The lower minimum is calculated as:

$$\text{Jaccard Minimum Value} = \begin{cases} \frac{x - (N - y)}{N} & \text{when } x + y \geq N \\ .00 & \text{when } x + y < N \end{cases} \tag{1}$$

where x and y are the frequency of occurrence of the two variables, and N is the number of cases within the sample.

[3] Sturidsson, Långström, Grann, Sjöstedt, Åsgård, and Aghede (2006) appear to provide evidence of a non-confirmatory pattern of frequencies across the variable configuration. However, they include in their analysis a variable "Victim Participation Verbal" that did not occur in any of the cases (see Table 1, p. 226). Jaccard coefficient cannot be calculated in this circumstance, suggesting that their analysis incorporated additional criteria that may have distorted the result.

[4] Equivalent results were obtained from a Monotoncity analyses in which this assumption was not made.

[5] This number was calculated by taking the Standard Deviations for each mean (i.e., the 100 data sets) and multiplying by 1.96, which provides the value above (and below) zero that is expected to occur 5% of the time (i.e., $p < .05$).

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Table 1

Mean Number of Variables Not Fitting a Visual Interpretation of the Modular Facet as a Function of Form of Data

Paper Type	Paper	Form of Data			Misplaced Variables	
		Variables	Cases	Highest Freq. (%)	Mean (SD)	Proportion of Variables
Original	Canter & Heritage (1990)	33	66	55 (83)	2.15 (.91)	.065
	Canter & Fritzon (1998)	42	175	147 (84)	2.21 (0.97)	.052
	Canter, Bennell, Alison & Reddy (2003)	27	112	92 (82)	1.69 (1.01)	.062
Adjusted	Canter & Heritage (1990)	33	66	66 (100)	2.00 (.85)	.061
	Canter & Fritzon (1998)	42	175	88 (50)	3.56 (1.11)	.08
	Canter, Bennell, Alison & Reddy (2003)	27	112	28 (25)	3.07 (1.19)	.11

Figure Captions

Figure 1. Schematic representation of a radex model of criminal behaviour.

Figure 2. Mean *v* coefficient for the evaluation data as a function of the highest frequency of occurrence of the variables.

Figure 3. Mean monotonicity coefficient for the evaluation data as a function of the highest frequency of occurrence of the variables.

Figure 4. Mean monotonicity coefficient for the artificial data as a function of the highest frequency of occurrence, number of variables, and skew of frequency distribution.

Figure 1

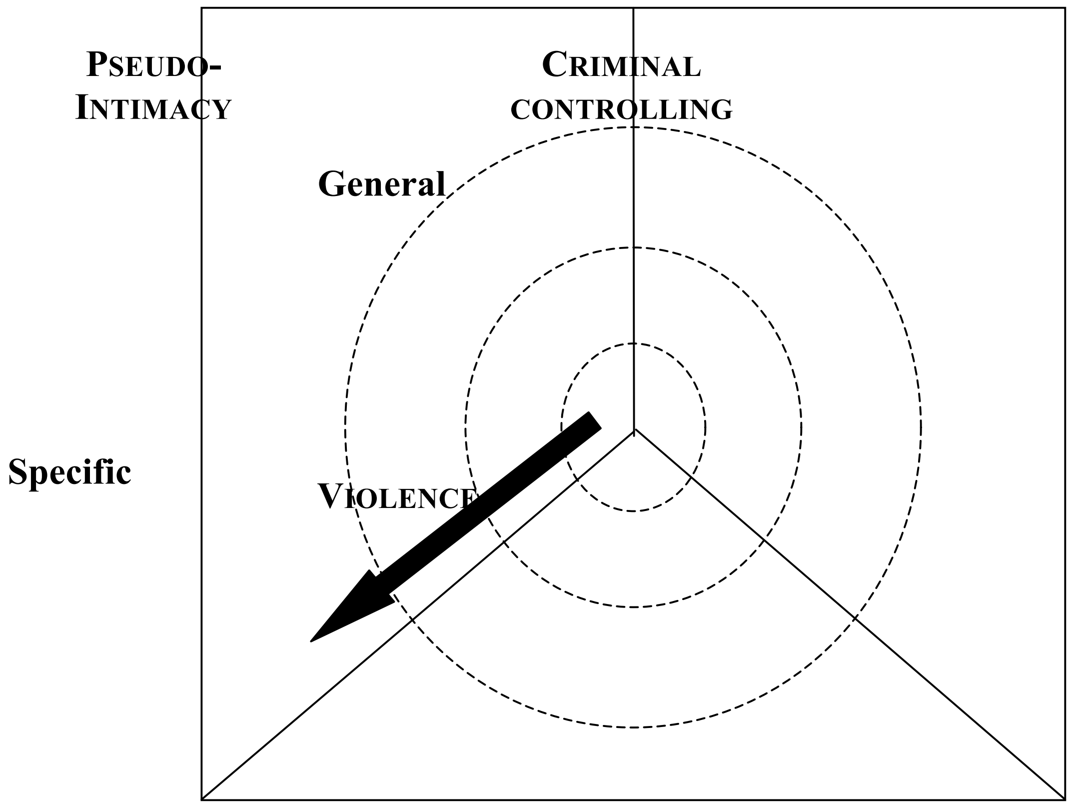

PSEUDO-
INTIMACY
CRIMINAL
CONTROLLING
General
Specific
VIOLENCE

Figure 2

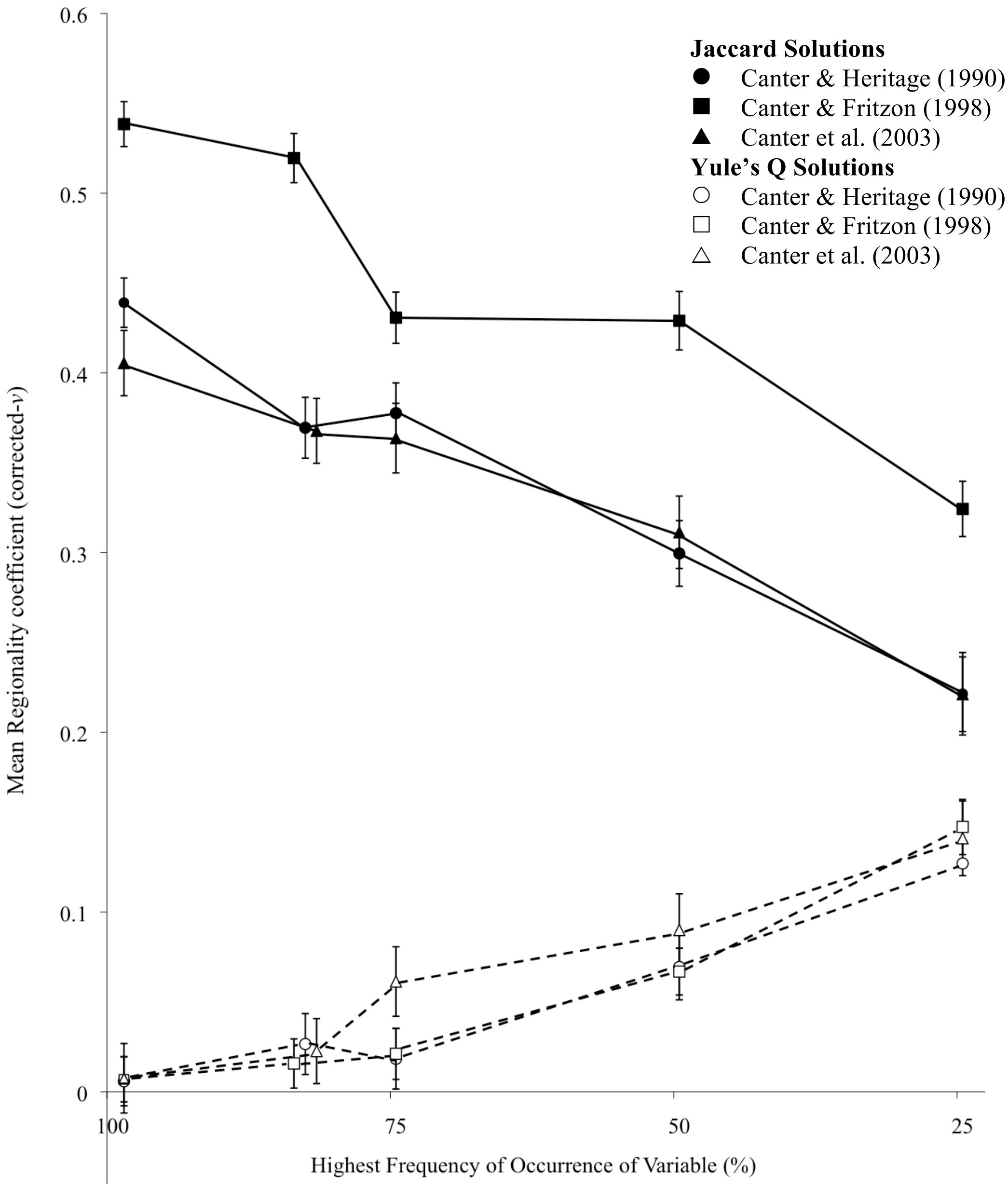

Jaccard Solutions
Canter & Heritage (1990)
Canter & Fritzon (1998)
Canter et al. (2003)
Yule's Q Solutions
Canter & Heritage (1990)
Canter & Fritzon (1998)
Canter et al. (2003)
Mean Regionality coefficient (corrected-v)
0.6
0.5
0.4
0.3
0.2
0.1
0
100
75
50
25
Highest Frequency of Occurrence of Variable (%)

Figure 3

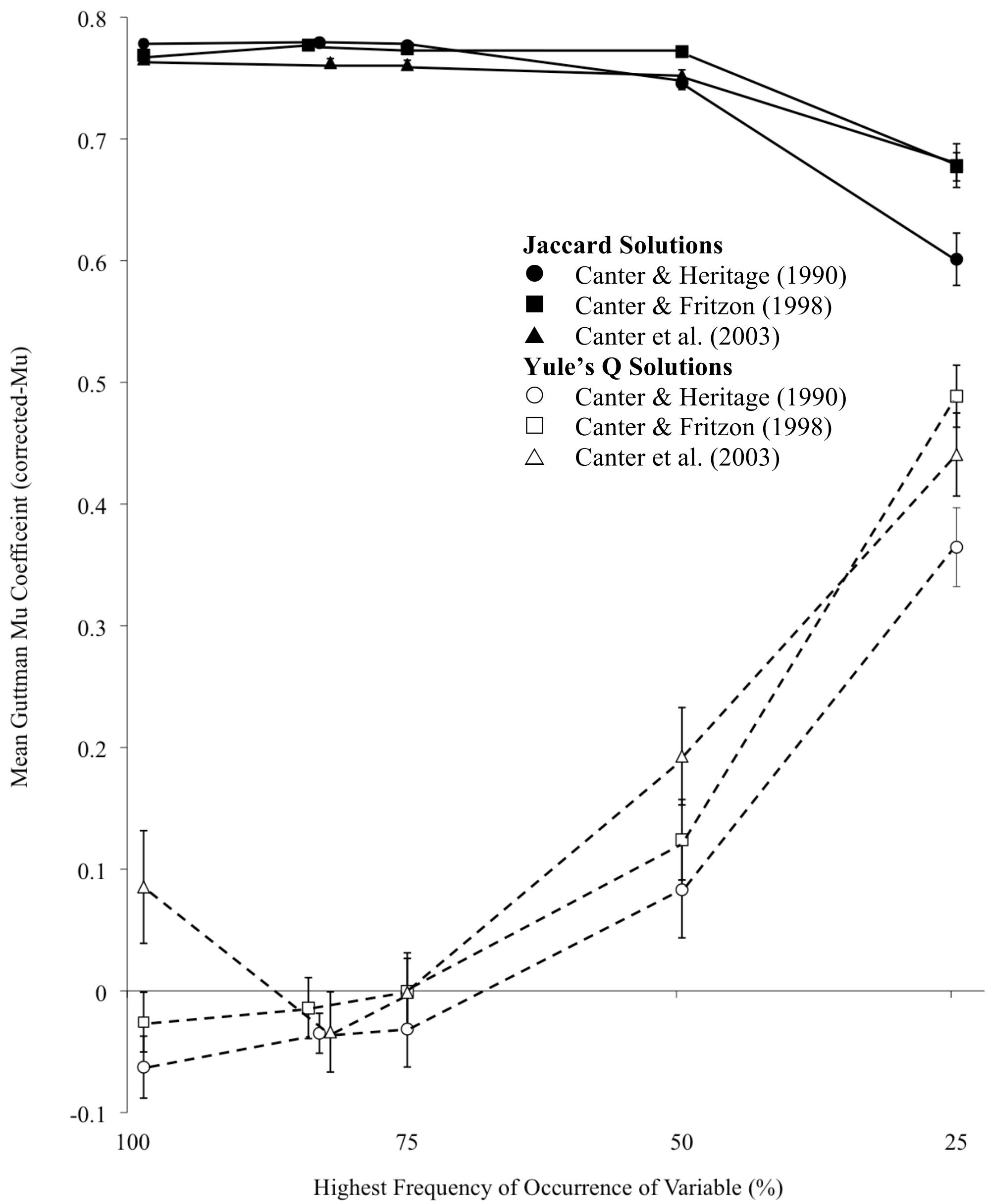
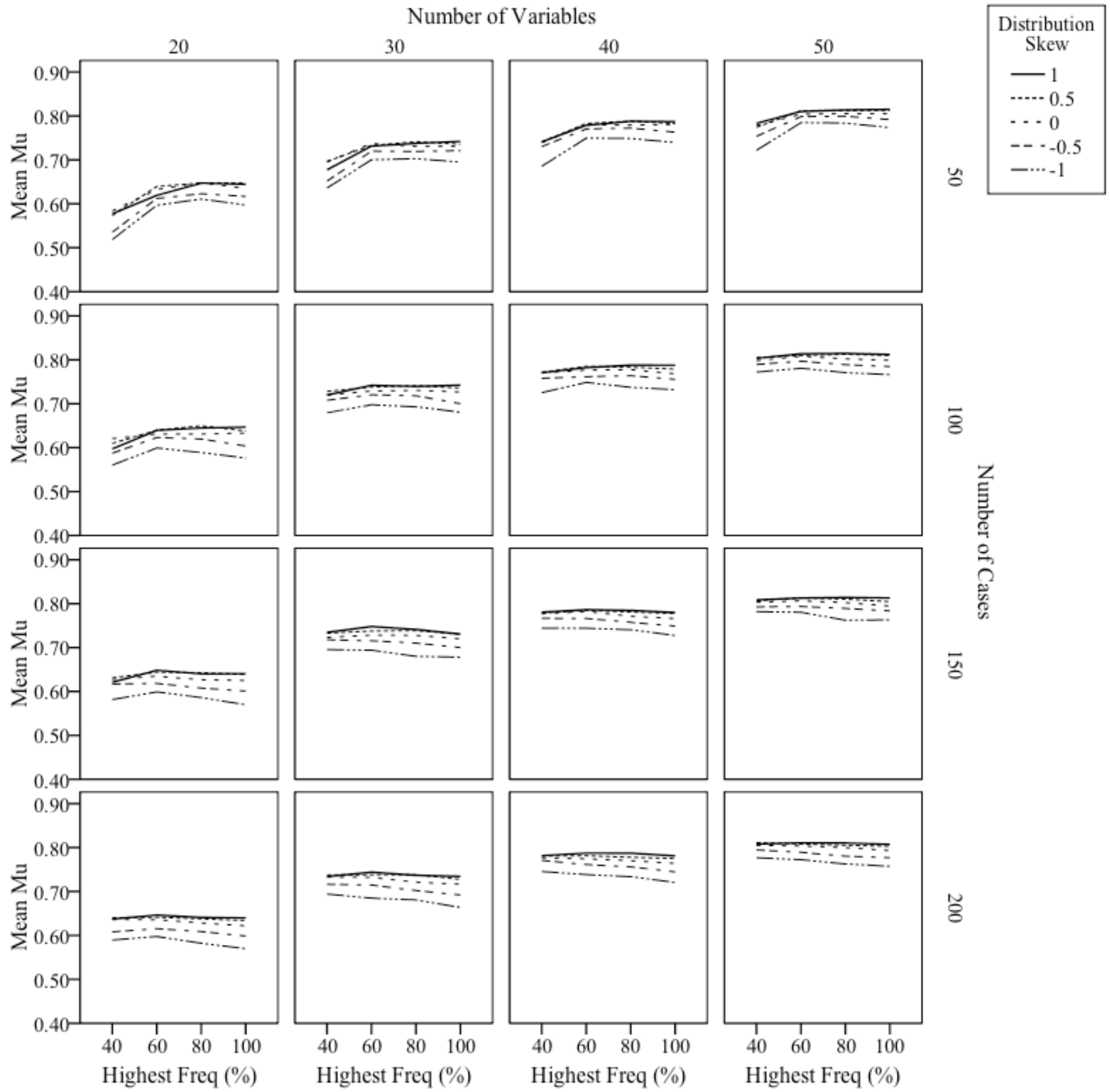

Mean Guttman Mu Coefficeint (corrected-Mu)
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-0.1
100
75
50
25
Highest Frequency of Occurrence of Variable (%)
Jaccard Solutions
Canter & Heritage (1990)
Canter & Fritzon (1998)
Canter et al. (2003)
Yule's Q Solutions
Canter & Heritage (1990)
Canter & Fritzon (1998)
Canter et al. (2003)

Figure 4


Number of Variables
20
30
40
50
Distribution Skew
1
0.5
0
-0.5
-1
Mean Mu
0.90
0.80
0.70
0.60
0.50
0.40
50
100
150
200
Number of Cases
40 60 80 100
Highest Freq (%)